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State-Space Modeling of Aerodynamic Forces on Plate Using Singular Value Decomposition

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Nomenclature

A, B, C, D	= state-space matrices
$\hat{A}, \hat{B}, \hat{C}, \hat{D}$	= reduced-order state-space matrices
D_{mn}	= aerodynamic influence coefficient
H	= Hankel matrix
$H_{mn}(t)$	= aerodynamic influence function
$h(kT)$	= matrix impulse response
$I_{mn}(t)$	= aerodynamic influence function
k	= discrete time index
$Q_n(t)$	= generalized force
$q_n(t)$	= generalized coordinate
S_{mn}	= aerodynamic influence coefficient
T	= discrete time step
t	= time
U	= eigenvector matrix of HH^T
u	= state-space input vector
V	= eigenvector matrix of H^TH
x	= state variable vector
y	= state-space output vector
ε	= infinity norm error bound
Σ	= singular value matrix
σ_i, ϵ_i	= singular values
τ	= integration dummy variable

Introduction

THE study of aeroelastic panels has received much attention in the literature lately, primarily focusing on the effects of composite materials[1] and the application of active control to flutter instabilities.[2–6] However, most of these studies employ piston theory or quasisteady theory as the working aerodynamic model. Use of these theories is not without good reason because modeling full potential flow aerodynamic forces on a plate can be difficult. Piston and quasisteady theory can also be readily cast in state variable form, which is a requirement for the application of modern and optimal control theory. The drawback of these low-order aerodynamic theories is that they are valid only for low reduced frequencies and high supersonic Mach numbers and are invalid for subsonic flow.

The full potential flow analysis published by Dowell[7,8] is valid at all subsonic and supersonic Mach numbers. However, the time domain convolution form of the aerodynamic forces is not conducive to use with modern control theory. To date only Frampton et al.[4,5] have employed full potential flow aerodynamics in the state variable modeling of the unsteady aerodynamic forces on a panel and subsequent application of modern control theory. This work began with the aerodynamic force expression provided by Dowell[7,8] and applied Prony's method to represent the aerodynamics in state variable form.[4] Although this proved to be an effective modeling method, it did not result in common system poles describing global dynamics. It is assumed that a linear aerodynamic system will have common poles (or eigenvalues) that characterize the system dynamics.[9–11] The application of Prony's method employed by Frampton et al.[4] calculated each path through the aerodynamic system independently, resulting in uncommon system poles. The resulting system had many approximately equal poles that should have been identical. Therefore, the system was of a much greater order than necessary.

Many investigations have been published that present methods for approximating the aerodynamic forces on wings and sections. Most of these investigations perform the approximation in the frequency domain based on numerically calculated transfer functions.[12] Padé approximations, which are very similar in approach to Prony's method, are the most frequently employed.[13–15] Roger[11] recognized that these techniques did not result in common poles and published a modified Padé approximation method that resulted in common poles. Each of these techniques amounts to system identification based on numerically calculated unsteady forces. To date, however, only Frampton et al.[4] have attempted to do this for the aerodynamic forces on a plate.

Recent work by Hall et al.[9] and Romanowski and Dowell[10] demonstrates that accurate reduced order aerodynamic models with common global poles can be created from high-order numerically based models. This led the authors to identify a similar aerodynamic system approximation technique that computes all paths through the aerodynamic system simultaneously based on a high-order numeric model, thus resulting in common system poles. The proposed technique results in a significant model reduction compared to the previously used Prony method.[4]

In this note, a method of approximating the aerodynamic forces on a plate subjected to full potential flow aerodynamic loading is presented. This technique is based on a singular value decomposition (SVD) technique, developed by Kung,[16–18] of the time domain aerodynamic forces as presented by Dowell.[7,8] The benefit of this approach over the previously used Prony method is that common paths through the aerodynamic system are computed simultaneously, resulting in common system poles. This results in a system of lower order with comparable accuracy. Additionally, the aerodynamic system is provided in state variable form, which permits the utilization of modern and robust control techniques. The cost of the SVD technique relative to Prony's method is increased up-front computational effort.

SVD

A general formulation for the SVD technique is presented in this section. A discussion of the specific application of this method will follow.

Consider a discrete time state-space realization of the form

$$x[(k+1)T] = Ax(kT) + Bu(kT) \tag{1}$$

$$y(kT) = Cx(kT) + Du(kT) \tag{2}$$

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where T is the time increment and k is the time index. The system $(\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{C}, \boldsymbol{D})$ has n states $\boldsymbol{x}$, p inputs $\boldsymbol{u}$, and m outputs $\boldsymbol{y}$. The matrix valued impulse response for the system is defined as

$$h(kT) = \begin{cases} \boldsymbol{D} & \text{for} \quad k = 0 \\ \boldsymbol{C}\boldsymbol{A}^{k-1}\boldsymbol{B} & \text{for} \quad k = 1, 2, \ldots, N+1 \\ \boldsymbol{0} & \text{for} \quad k > N+1 \end{cases} \tag{3}$$

A block Hankel matrix can be formed from the matrix valued impulse response as

$$\boldsymbol{H} = \begin{bmatrix} \boldsymbol{h}(T) & \boldsymbol{h}(2T) & \boldsymbol{h}(3T) & \cdots & \boldsymbol{h}\{(N+1)T\} \\ \boldsymbol{h}(2T) & \boldsymbol{h}(3T) & \boldsymbol{h}(4T) & \cdots & \boldsymbol{0} \\ \boldsymbol{h}(3T) & \boldsymbol{h}(4T) & \boldsymbol{h}(5T) & \cdots & \boldsymbol{0} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{h}\{(N+1)T\} & \boldsymbol{0} & \boldsymbol{0} & \cdots & \boldsymbol{0} \end{bmatrix} \tag{4}$$

where $\boldsymbol{H}$ is of dimension $m(N+1) \times p(N+1)$. The first step toward obtaining a state-space realization of the system that is characterized by the matrix valued impulse response of Eq. (3) is to perform a singular value decomposition of the block Hankel matrix. The decomposition has the form

$$\boldsymbol{H} = U \Sigma V^T = \boldsymbol{U}\boldsymbol{V}^T \tag{5}$$

where U is an $m(N+1) \times (N+1)$ orthogonal matrix of the eigenvectors of $\boldsymbol{H}\boldsymbol{H}^T$, V is a $p(N+1) \times (N+1)$ orthogonal matrix of the eigenvectors of $\boldsymbol{H}^T\boldsymbol{H}$, and the singular value matrix is

$$\Sigma = \text{diag}[\sigma_1, \sigma_2, \ldots, \sigma_r, \epsilon_{r+1}, \epsilon_{r+2}, \ldots, \epsilon_{N+1}] \tag{6}$$

with the singular values in descending order such that

$$\sigma_1 \ge \sigma_2 \ge \cdots \sigma_r \ge \epsilon_{r+1} \ge \epsilon_{r+2} \ge \cdots \ge \epsilon_{N+1} \tag{7}$$

and

$$\boldsymbol{U} = U \Sigma^{\frac{1}{2}} \tag{8}$$

$$\boldsymbol{V}^T = \Sigma^{\frac{1}{2}} V^T \tag{9}$$

If the matrix $\boldsymbol{H}$ has a rank of r, then the last $N - r$ singular values $(\epsilon_i : i = r+1, r+2, \ldots, N+1)$ would be zero. If the singular values ϵ_i are not zero but are much less than the singular values σ_i, then the matrix is very near to rank r. In this case, the singular values ϵ_i represent computational noise in the impulse responses and modes with relatively small contribution. This property is taken advantage of in selecting the order of the final model.

A reduced-order realization $(\hat{\boldsymbol{A}}, \hat{\boldsymbol{B}}, \hat{\boldsymbol{C}}, \hat{\boldsymbol{D}})$ of order r, which omits the excess modes, can be had by partitioning the SVD as follows:

$$\boldsymbol{H} = \begin{bmatrix} \boldsymbol{U}_{1,1} & \boldsymbol{U}_{1,2} \\ \boldsymbol{U}_{2,1} & \boldsymbol{U}_{2,2} \\ \vdots & \vdots \\ \boldsymbol{U}_{N,1} & \boldsymbol{U}_{N,2} \\ \boldsymbol{U}_{N+1,1} & \boldsymbol{U}_{N+1,2} \end{bmatrix} \begin{bmatrix} \boldsymbol{V}_{1,1}^T & \boldsymbol{V}_{2,1}^T & \cdots & \boldsymbol{V}_{N,1}^T & \boldsymbol{V}_{N+1,1}^T \\ \boldsymbol{V}_{1,2}^T & \boldsymbol{V}_{2,2}^T & \cdots & \boldsymbol{V}_{N,2}^T & \boldsymbol{V}_{N+1,2}^T \end{bmatrix} \tag{10}$$

where the block matrices $\boldsymbol{U}_{i,1}; i = 1, 2, \ldots, N+1$ are $m \times r$, $\boldsymbol{U}_{i,2}; i = 1, 2, \ldots, N+1$ are $m \times (N+r-1)$, $\boldsymbol{V}_{i,1}; i = 1, 2, \ldots, N+1$ are $p \times r$ and $\boldsymbol{V}_{i,2}; i = 1, 2, \ldots, N+1$ are $p \times (N+r-1)$.

The reduced-order realization can be defined based on these partitions as follows:

$$\hat{\boldsymbol{A}} = \left(\begin{bmatrix} \boldsymbol{U}_{1,1} \\ \boldsymbol{U}_{2,1} \\ \vdots \\ \boldsymbol{U}_{N,1} \end{bmatrix}^T \begin{bmatrix} \boldsymbol{U}_{1,1} \\ \boldsymbol{U}_{2,1} \\ \vdots \\ \boldsymbol{U}_{N,1} \end{bmatrix} \right)^{-1} \begin{bmatrix} \boldsymbol{U}_{1,1} \\ \boldsymbol{U}_{2,1} \\ \vdots \\ \boldsymbol{U}_{N,1} \end{bmatrix}^T \begin{bmatrix} \boldsymbol{U}_{2,1} \\ \boldsymbol{U}_{3,1} \\ \vdots \\ \boldsymbol{U}_{N+1,1} \end{bmatrix} \tag{11a}$$

$$\hat{\boldsymbol{B}} = \boldsymbol{V}_{1,1}^T \tag{11b}$$

$$\hat{\boldsymbol{C}} = \boldsymbol{U}_{1,1} \tag{11c}$$

and

$$\hat{\boldsymbol{D}} = \boldsymbol{h}(0) \tag{11d}$$

The infinity norm error bound of the reduced model relative to the model retaining all singular values can be expressed as a sum of the discarded singular values as follows:

$$\varepsilon \le 2 \sum_{i=r+1}^{N+1} \epsilon_i \tag{12}$$

Proofs of the preceding realization along with proofs of stability, etc., can be found in Refs. 16 and 17. A proof of the error bound can be found in Ref. 18.

Application of SVD to an Aeroelastic Plate

The SVD modeling method described in the preceding section can be used for any system that is described as a time domain convolution. The example used here models the aerodynamic forces on a plate as described by Dowell[7,8] and more recently by Frampton et al.[4] The method can also be used to model the aerodynamic forces on a wing as described by the Duhamel integrals in Ref. 19.

Aerodynamic Forces on a Plate

As noted previously, most recent efforts toward modeling aeroelastic panels use quasisteady or piston theory as the aerodynamic model. These theories are limited to supersonic flow and low-frequency response. Using full-potential flow aerodynamic modeling, although much more difficult, has several advantages. First, full-potential flow is accurate for all frequencies and Mach numbers including subsonic flow. This includes the phenomenon of subsonic divergence or buckling and single mode flutter (also known as negative damping flutter). A further advantage of full-potential flow modeling is that results can be used to study the effects of flow on acoustic radiation from plates.[20]

The generalized forces on the plate, $Q_n(t)$, are found by solving the full-potential flow partial differential equation describing the velocity potential in an inviscid, irrotational fluid moving parallel to the x axis subject to the boundary conditions for a panel embedded in an infinite baffle. The solution is obtained through a Laplace–Fourier transform technique as described by Dowell.[7,8]

The solution yields the generalized aerodynamic forces on the plate, which are given here as

$$Q_n(t) = \sum_{m=1}^{J} Q_{mn}(t) \tag{13}$$

where J is the number of plate modes and $Q_{mn}(t)$ is the generalized force on the nth plate mode due to motion of the mth plate mode such that

$$Q_{mn}(t) = q_m(t) S_{mn} + \dot{q}_m(t) D_{mn} + \int_0^t q_m(\tau) H_{mn}(t-\tau)\, d\tau + \int_0^t \dot{q}_m(\tau) I_{mn}(t-\tau)\, d\tau \tag{14}$$

where $q_m(t)$ and $\dot{q}_m(t)$ are the plate generalized coordinate and plate generalized velocity, respectively. Definitions of the influence coefficients S_{mn} and D_{mn} and influence functions $H_{mn}(t)$ and $I_{mn}(t)$ may be found in Refs. 4, 7, and 8. Note that the influence functions are the impulse responses that characterize the aerodynamic system.

Approximation of the Aerodynamic Forces

The first step toward approximating the aerodynamic forces on a plate is to calculate the influence functions $H_{mn}(t)$ and $I_{mn}(t)$ through numeric integration.[4,7,8] This provides the aerodynamic influence functions at discrete times $t = kT$. Next the matrix valued impulse response for the aerodynamic system must be constructed. The final aerodynamic system will receive the plate generalized coordinates and velocities as inputs and generate as outputs the aerodynamic generalized forces. Therefore, the desired aerodynamic system input and output vectors, for a plate having J modes, are

$$\boldsymbol{u} = [q_1 \quad q_2 \quad \cdots \quad q_J \quad \dot{q}_1 \quad \dot{q}_2 \quad \cdots \quad \dot{q}_J]^T \tag{15}$$

and

$$y = [Q_1 \quad Q_2 \quad \cdots \quad Q_J]^T \tag{16}$$

respectively. Considering Eq. (3) and the desired ordering of the inputs and outputs leads to a matrix valued impulse response of

$$\boldsymbol{h}(kT) = \begin{cases} \begin{bmatrix} S_{11} & S_{21} & \cdots & S_{J1} & D_{11} & D_{21} & \cdots & D_{J1} \\ S_{12} & S_{22} & \cdots & S_{J2} & D_{12} & D_{22} & \cdots & D_{J2} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ S_{1J} & S_{2J} & \cdots & S_{JJ} & D_{1J} & D_{2J} & \cdots & D_{JJ} \end{bmatrix} & \text{for} \quad k = 0 \\ \begin{bmatrix} H_{11}(kT) & H_{21}(kT) & \cdots & H_{J1}(kT) & I_{11}(kT) & I_{21}(kT) & \cdots & I_{J1}(kT) \\ H_{12}(kT) & H_{22}(kT) & \cdots & H_{J2}(kT) & I_{12}(kT) & I_{22}(kT) & \cdots & I_{J2}(kT) \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ H_{1J}(kT) & H_{2J}(kT) & \cdots & H_{JJ}(kT) & I_{1J}(kT) & I_{2J}(kT) & \cdots & I_{JJ}(kT) \end{bmatrix} & \text{for} \quad k = 1, 2, \ldots \\ \boldsymbol{0} & \text{for} \quad k > N + 1 \end{cases} \tag{17}$$

where the influence coefficients are incorporated as the instantaneous terms in the impulse response. At this point, the block Hankel matrix of Eq. (4) can be constructed from the impulse response matrix of Eq. (17). Then the realization of Eqs. (11a–11d) can be formed. Note that the states of the resulting system have no physical significance but are of mathematical construct. The states are, however, related to the unsteady aerodynamic pressure on the plate through the $\hat{\boldsymbol{C}}$ matrix.

As an example, two realizations were computed for a plate having the first four chord wise modes [i.e., (1, 1), (2, 1), (3, 1), and (4, 1) modes] and a length to width ratio of a:b = 1.0 at a Mach number of 2.0. One system had an allowable error bound of $\varepsilon \leq 1.0$ and the second had $\varepsilon \leq 0.3$. The resulting systems had 11 and 21 states for the $\varepsilon \leq 1.0$ and $\varepsilon \leq 0.3$, respectively. The model presented in Ref. 4 required approximately 60 states to achieve an accuracy similar to that of the $\varepsilon \leq 0.3$ systems. Note that the SVD system, which has common global poles, required one-third as many states as the system created by Prony's method. Although the up-front computational cost associated with the SVD technique is large, the investment is well worth the reduced time spent analyzing the final system.

Two realizations were also constructed at a Mach number of 1.4 with the same error criterion. These resulted in 13 and 26 state systems. The model presented in Ref. 4 with accuracy similar to the $\varepsilon \leq 0.3$ systems required over 80 states.

A comparison of the generalized force on the (1, 1) mode resulting from a step change in the (1, 1) mode is shown in Fig. 1 vs nondimensional time, $s = tU/a$, where U and a are the flow speed and panel chord, respectively. Two reduced-order models and the force calculated directly from Eq. (14) are shown. Note that the higher-order system, which resulted from a lower allowable error bound, follows the directly computed results well. A further increase in the system order would result in a response that is graphically indistinguishable from the directly computed result.

A similar comparison is shown in Fig. 2 for the generalized force on the (1, 1) mode due to a step change in the (2, 1) mode. Note the instantaneous ($s = 0^+$) response that results from the influence coefficients. The instantaneous response is equivalent to piston theory. All of these results compare well with those found by Dowell[7] and Frampton et al.[4]

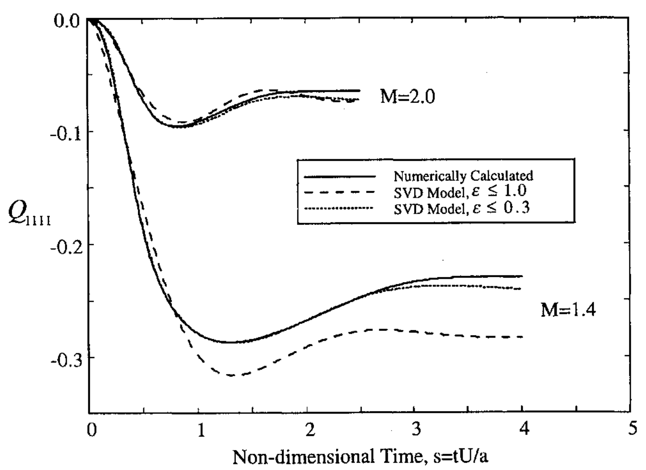

Fig. 1 Generalized force on the (1, 1) mode due to a step change in the (1, 1) generalized coordinate for numerically calculated and two reduced-order systems.

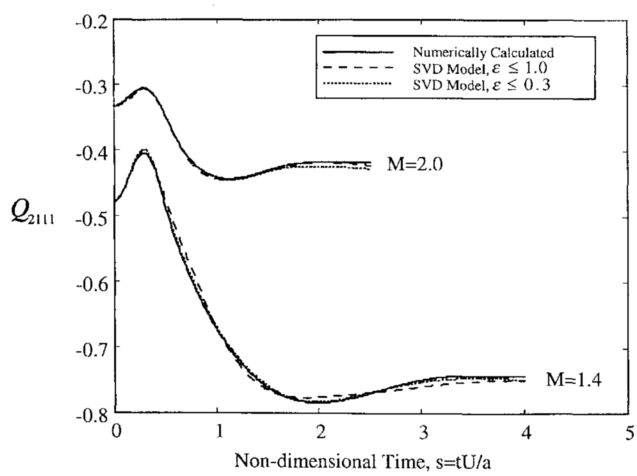

Fig. 2 Generalized force on the (1, 1) mode due to a step change in the (2, 1) mode generalized coordinate for numerically calculated and two reduced-order systems.

Conclusions

A technique for state-space modeling of full potential flow aerodynamic forces on a plate has been presented. This technique employs the singular value decomposition of a block Hankel matrix constructed from the aerodynamic matrix valued impulse response resulting in a system that has common global poles. These common poles result in a system of much lower order than previous techniques having similar accuracy. Comparison between the SVD realization and numerically computed results demonstrates the accuracy of this approach. Several advantages of using full-potential flow aerodynamics were also discussed, including validity over all Mach numbers and frequencies and the ability to be used with modern and optimal control theories.

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Modeling Influences of Inlet Swirl Profiles on Dump Combustor Flows

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Introduction

A CONFINED jet exiting into a sudden expansion, in which reverse flow occurs immediately downstream of the sudden expansion, is encountered in many practical combustion systems. Because of its importance, numerous numerical studies have been made by various researchers to study the effects of swirl in a variety of combustor geometries.[1–4] The studies demonstrate the superiority of stress closures over the k-ϵ model in the prediction of swirling flows, though the merits of alternative stress model variants differ at different swirl levels and swirler types.

Although cubic[2] and quadratic[3] pressure-strain models have been applied to further explore the effects of nonlinear pressure-strain models on predicting free swirling flows, the majority of the confined swirling flow predictions are limited to the linear models. The present research aims at investigating the capability of variants of Reynolds stress turbulence models, linear or quadratic pressure-strain models, on sudden-expanding-pipe geometry swirling flows. Flows with forced and free vortex swirl profiles at the inlet and swirl number of 0.4 form the basis of the investigations.

Turbulence Models

In the present application, turbulence is described either by the k-ϵ model or by Reynolds stress closures. The focal point of a Reynolds stress model is the pressure-strain term ϕ_{ij}, which identifies pressure/strain interaction and consists of three model components representing, respectively, return to isotropy ϕ_{ij1}, isotropization of mean-strain and turbulence correlation ϕ_{ij2}, and redistributive effects arising from wall reflection of pressure fluctuations ϕ_{ijw}.

Four variants of the linear form of the pressure-strain models were investigated. The first stress model closure variant (IPGL) adopted here is that of Gibson and Launder.[5] A variant (IPCM) of the preceding closure, proposed by Fu et al.,[2] includes the convection tensor C_{ij} in ϕ_{ij2} to arrive at the coordinate invariant, objective model form. The second variant (IPGY), proposed by Gibson and Younis,[1] modifies the coefficients C_1 and C_2 in the IPGL model by taking the values as 3.0 and 0.3, respectively. A third variant (LRR) adopted is that proposed by Launder et al.[6] Instead of the linear pressure-strain model, a variant (SSG) proposed by Speziale et al.[7] employs quadratic form of the pressure-strain process. This model does not include explicitly wall reflection terms.

Computational Model

The present numerical procedure[4] solves discretized versions of all equations on a staggered finite volume arrangement, incorporating the SIMPLE pressure-correction algorithm[8] and the QUICK scheme for convective fluxes.[9] The geometry of the dump combustor is a sudden-expanding pipe and the expansion ratio based on the ratio of the radii is 1.5. The step height H is defined as the difference of the radii of the sudden-expanding pipe. The coordinates X and R are denoted as the axial and radial directions, respectively; $X/H = 0$ represents the location of the expansion, and $R/H = 0$

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